Locallity derived from diffeomorphism invariance, using fiber bundle.

H.B. Nielsen(Copenhagen), Astri Kleppe ...

Bled , July, 2019

[4] *The Niels Bohr Institute, Copenhagen*
H.B. Nielsen presents the talk.

Why do we have locallity ?

How little do we have to assume to derive locallity ?
We here take locallity to mean that the Lagrange which we could use - so a kind of effective Lagrangian - is one of the usual integral over space-time form

$$S[\phi, \psi, ...] \;=\; \int \mathcal{L}\sqrt{g}\, d^d x. \tag{1}$$

Locality concept

In a generic physical model, the property of locality is usually taken for granted. Perhaps that is the reason why it the actual meaning of locality so seldom is discussed at great length. We usually think of locality in terms of information being localized, propagating from one spacetime point to another by at most the speed of light. Another way of expressing it, is that all cause-and-effect relations are limited by the speed of light. Interactions are assumed to be local, taking place in one spacetime point, implying that one spatio-temporal site is assigned to each degree of freedom.

Concept of locality (continued)

We know that there is locality when an experiment in one place has no immediate influence on an experiment in an other place, this is also true for effects like the butterfly effect, because they take time.

Nonlocality, on the other hand, refers to a situation where information instantaneously spreads out over a large distance. In a nonlocal theory, the degrees of freedom are functions of more than one spacetime point. This allows for making predictions in a noncausal way, i.e. to get information about parts of the Universe that are at a spacelike interval from ourselves.

Locality Empirically true.

At our everyday, classical level, reality seems convincingly local, which is reflected in that the Laws of Nature - the equations of physics - are local and the continuity equation tells that there are no jumps!

Random Dynamics = Laws of Nature Emerge Automaticaly

In the Random Dynamics approach [1], locality is however not perceived as fundamental. The reason is that the primal Random Dynamics world machinery is a very general, random mathematical structure which merely contains non-identical elements and some set-theoretical notions. From this "world machinery" differentiability and a concept of distance (geometry) are to be derived, as well as as well as space and time [2], Lorentz invariance, diffeomorphism symmetry [3], locality, and eventually all other physical concepts. But even after locality is derived, some smeared out left-over nonlocal effects remain, showing up in coupling constants (which feel an average over spacetime, and also depend on such averages). This remaining (mild) nonlocality is moreover supported by the Multiple Point Principle (MPP) [4].

Deriving Locality

In the present work we attempt to derive locality from an analytic and diffeomorphic symmetric action, using fiber bundle formalism. We start with an action $S[\varphi]$ and

- a fiber bundle of dimension ≥ 4
- the conviction that we can get genuine locality (not super locality)
- along $p = 4$

  dimensional p-surfaces

When you have a field configuration (cross section) on your fiber bundle (a compact fiber bundle that you can integrate over), it is in the spirit of fields and cross sections that you can only make various local functions of them.

Integral form

These fields must be integrated over, because for a field on a manifold $\mathcal{M}$, a value in one single point has no meaning (because it is of Lebesgue measure $= 0$), in the sense that in a continuous field a given point can always be replaced by some small integral piece. The integration is taken over such small integral pieces, and the final, generic action is some complicated combination of these integrals. If you have diffeomorphism symmetry, you cannot have boundaries on your integrals, so every integral must be taken over the whole space (which is assumed to be a connected manifold). Our action must is thus a function of integrals over the entire space.

We shall derive Action to be of Integral form Effectively

The locality that we want to derive comes about by formulating the action as an integral over space-time of a Lagrangian density

$$\mathcal{L}(\varphi(\mathrm{x}), \partial\varphi(\mathrm{x})/\partial\mathrm{x})$$

that only depends on the fields - such as φ - and their (up to finite order) derivatives taken with values of the space-time point x.
We have an action $S[\phi]$, where the function ϕ is defined over the manifold $\mathcal{M}$. By loosely identifying a point on $\mathcal{M}$ with its coordinates x_j, the summing over the various j=1,2,...,N becomes replaced by integrals over the coordinate variable sets on the manifold.

Taylor expansion of Functional

The Taylor expansion of the functional for a single function $\phi(\mathrm{x})$ is defined as

$$S[\phi] = S[\phi = 0] + \int \frac{\delta S}{\delta\phi(\mathrm{x})}\phi(\mathrm{x}) d^d x + \frac{1}{2!}\int\int \frac{\delta^2 S}{\delta\phi(\mathrm{x})\delta\phi(y)}\delta\phi(\mathrm{x})\delta\phi(y) d^d \mathrm{x}$$

$$= \sum_n \frac{1}{n!}\frac{\delta^n S}{\delta\phi(\mathrm{x}_1)\cdots\delta\phi(\mathrm{x}_n)}|_{(\phi=0)}\phi(\mathrm{x}_1)\cdots\phi(\mathrm{x}_n) d^d \mathrm{x}_1\cdots d^d \mathrm{x}_n. \quad (2)$$

First step: Keep d-volume fixed

Initially we only consider diffeomorphism transformations which leave the local d-volume invariant, i.e. a subset of diffeomorphism transformations $\mathrm{x}'(\mathrm{x})$ such that

$$det\left(\frac{\partial \mathrm{x}^{\mu}(\mathrm{x})}{\partial \mathrm{x}^{\nu}}\right) = 1, \quad (3)$$

which means that the d-volume of spacetime

$$\epsilon_{\mu\nu\cdots\tau} d\mathrm{x}^{\mu} d\mathrm{x}^{\nu} \cdots d\mathrm{x}^{\tau}$$

remains invariant under this subset of diffeomorphism transformations.

Second step: Allowing also d-volume non-conserving diffeomorphisms

In the second step, we include ("pseudoscalar") fields that have other tranformation properties under diffeomorphism transformations than mere scalars. Such a "pseudoscalar" field $P(\mathrm{x})$ transforms as

$$P(\mathrm{x}) \rightarrow P(\mathrm{x}')det\left(\frac{\partial \mathrm{x}_{\mu}(\mathrm{x})}{\partial \mathrm{x}_{\nu}}\right) \tag{4}$$

Imposing Diffeomorphism Symmetry on Taylor Expansion

Imposing the diffeomorphism symmetry coordinate shift $x \to \mathrm{x}'$, preserving the integral $d^d x = d^d \mathrm{x}'$ on the Taylor expansion

$$S[\phi] = \sum_{n=0,1,..} \phi(\mathrm{x}_1) \cdots \phi(\mathrm{x}_n) d^d \mathrm{x}_1 \cdots d^d \mathrm{x}_n \tag{5}$$

leads to the requirement that the coefficients, i.e. the derivatives

$$\frac{\delta S}{\delta\phi(\mathrm{x}_1) \cdots \delta\phi(\mathrm{x}_n)}$$

must be invariant in a similar way under diffeomorphism transformations.

"Pseudoscalar" Lagrangian density

With the condition of diffeomorhism invariance, these integrals must be pseudoscalar (i.e. transform as $\sqrt{g}$), resulting in an action which by definition is pseudoscalar, that is, a function (not a functional) of all the various pseudoscalar terms that we can construct, and we can make differentiation through all these pseudoscalar contributions.

The action is however not the main point: what matters is the equation of motion; the most important application of the action/Lagrangian is to get the equation of motion by the Euler-Lagrange equation, by differentiating our action/Lagrangian functionally with respect to a point on the manifold, and set it $= 0$.

Functional derivatives and Taylor expansion.

The functional derivative of S is a partial derivative of S with respect to all the different integrals over the whole space summed over, multiplied by the functional derivatives of the latter.

$$\frac{\delta[S]}{\delta\xi(\mathrm{x})} = \sum_k \frac{\partial S}{\partial V^k}\frac{\delta V^k}{\delta\xi(\mathrm{x})} = \frac{\delta S_{eff}}{\delta\xi(\mathrm{x})} \tag{6}$$

where

$$S_{eff} = \int_M \sum_k \frac{\partial S}{\partial V^k}\mathcal{P}_k(\mathrm{x})d^4\mathrm{x}, \tag{7}$$

and $\mathcal{P}_k(m) \approx \mathcal{L}_k(\mathrm{x})\sqrt{g(\mathrm{x})} = \mathcal{L}_k(m)\sqrt{g(m)}$ are pseudoscalars, V^k are the integrals

$$V^k = \int \mathcal{P}_k(m)dm, \tag{8}$$

and the $m \in \mathcal{M}$ are spacetime points/events.

In local coordinates x^{μ} on $\mathcal{M}$, $\mathrm{x}^{\mu}(m)$ are the coordinates of the event m, and $d^4\mathrm{x}$ is a measure in the coordinates x^{μ}, such that $d^4\mathrm{x} = d\mathrm{x}^1 d\mathrm{x}^2 \cdots d\mathrm{x}^d \equiv dm$.

Fiber bundles

We define our fiber bundle as $\mathcal{F} = (E, \pi, \mathcal{M})$, where the manifold $\mathcal{M}$ is the base space, E is the total space of the fiber bundle, and $\pi : E \to \mathcal{M}$ is a projection of the bundle (trivially $E = \mathcal{M} \times \mathcal{F}$). We interpret the fiber bundle F as the set of field configurations (or cross sections), and assume that $E = \mathcal{M} \times \mathcal{F}$ is locally valid.

Deriving locality using fiber bundle formulation

Our ambition is to use fiber bundle notation to clarify the derivation of locality from an analytic, diffeomorphic symmetric action using fiber bundle notation, with an extension to tensor bundles.
In the present work, by e. g. having a second order tensor field

$$\mathbf{g} = g^{\mu\nu} \frac{\partial}{\partial \mathrm{x}^{\mu}} \frac{\partial}{\partial \mathrm{x}^{\nu}},$$

we can formulate a theory with effective locality (and no super-locality).

Lack of complete derivation of locality

The point is that with the assumption of local diffeomorphism symmetry, the effective action comes out as

$$S_{eff} = \int \mathcal{L}_{(pseudoscalar)} dx^1 \wedge dx^2 \wedge \cdots dx^d \tag{9}$$

although with a very important detail: The coupling constants or coefficients become complicated integrals over the whole manifold/base space $\mathcal{M}$, i.e. over all spacetime including both future and past. So in this restricted sense the resulting theory is still non-local, although the non-locality only comes in via the coupling constants.

Analyticity of Functionals

We say that the functional S of $\varphi : \mathcal{M} \to E$ is "analytical" provided we have a convergent Taylor expansion

$$S[\varphi] = S[\varphi_0] + \int \frac{\delta S}{\delta \varphi^i(m)} (\varphi^i(m) - \varphi_0^i(m)) dm + \cdots \tag{10}$$

where

$$\frac{\delta S}{\delta \varphi(m)} \sim \frac{\delta S}{\delta \varphi^i(m)} d\varphi^i = dS \in [cotangent \ \ space \ \ for \ \ E] \tag{11}$$

Locally in E space you have coordinates, so define $\Delta\varphi^i(m)$ as

$$\Delta\varphi^i(m')|_{(at\ m)} = (\varphi^i(m') - \varphi_0^i(m'))\frac{\partial}{\partial\varphi^i(m)} \qquad (12)$$

which is a tangent vector, and $\delta S/\delta\varphi^i(m)$ is the basis of the tangent space.

With the coordinates $\mathrm{x}^\mu = (\mathrm{x}^1, \mathrm{x}^2, \cdots)$, the basis in the tangent space is

$$\frac{\partial}{\partial \mathrm{x}^\mu} = \tau^\mu,$$

and a tangent vector $t_\mu \tau^\mu \in T$, where T is a tangent vector space.

Tangent vector

A tangent vector field is given by a $t_\mu(\mathrm{x})$ with lower index, while a one-form field $o = o_\mu d\mathrm{x}^\mu$ is given by a lower index field o_μ, so $g^{\mu\nu}$ can be a field describing a ("tangent") tensor in a space with basis vectors

$$\frac{\partial}{\partial \mathrm{x}^\mu} \frac{\partial}{\partial \mathrm{x}^\nu}$$

Summary of Set up.

So we have:

1. A fiber bundle of dim ≥ 4
2. We hope to get genuine (i.e not super-) locality along $p = 4$ dimensional p-surfaces.
3. By a screw (or an even more chaotic arrangement of Nature) we get the $p = 4$ dimensional p-surfaces to almost cover the d-dimensional (base space) volume $\mathcal{M}$.
4. It is the 4-dimensional "wild" thread that is our spacetime: it is so to speak curled up in the $d > 4$ dimension.
5. The running of the thread of 3+1 dimensions is given by say the vierbein V_a^ν, which means a set of four tangent vectors

Vierbein...

It is specified that the $p = 4$ surfaces should run through the d-dimensional $\mathcal{M}$-space so as to have their 4-dimensional tangent vector space embedded (naturally) in the tangent space of $\mathcal{M}$ at the point m, in such a way that it is just the one that is spanned by the four tangent vectors $V_a = V_a^\mu \partial/\partial \mathrm{x}^\mu$.
We propose a "pseudoscalar" of the form $\mathcal{P}_1 = \xi_{d-4}\omega_4$, where ξ_{d-4} are suggested uninteresting for us.

In our earlier work, we assumed that our action was diffeomorphic symmetric and analytic. In the present article, we however relax the assumptions, by starting from an action functional which is diffeomorphic symmetric and continuous.

Now

$$\frac{\delta S}{\delta \varphi^i(m)} \Delta \varphi^i(m)|_{at\ m} = \frac{\delta S}{\delta \varphi^i(m)} d\varphi^i \Delta \varphi(m')|_{at\ m'} = dS(m)\Delta\varphi(m)|_{at\ m} \tag{13}$$

where

$$d\varphi^i \frac{\partial}{\partial \varphi^k(m)} = \delta^i_k,$$

In order to express this in the language of functionals, we expand S around φ_0:

$$S[\varphi] = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n S[\varphi_0]}{\partial \varphi^{i_1}(m_1) \partial \varphi^{i_2}(m_2) .. \partial \varphi^{i_n}(m_n)} \cdot \tag{14}$$

$$\cdot (\varphi^{i_1}(m_1) - \varphi_0^{i_1}(m_1))(\varphi^{i_2}(m_2) - \varphi_0^{i_2}(m_2)) \cdots (\varphi^{i_n}(m_n) - \varphi_0^{i_n}(m_n)) dm_n, \tag{15}$$

and then define a dual function to the tangent space vector, i.e. a covector, $\mathcal{D}\varphi^i(m)$, by using the tangent space basis vectors

$$\frac{\partial}{\partial \varphi^j(m')} \tag{16}$$

to define the number

$$\langle \mathcal{D}\varphi^i(m) | \frac{\partial}{\partial \varphi^j(m')} \rangle = \delta^i_j \delta(m - m') \tag{17}$$

One tests $\delta(m-m')$ by a test function $K(m)$,

$$K(m)\mathcal{D}\varphi^i(m)\frac{\partial}{\partial\varphi^i(m')}=\delta^i_jK(m') \qquad (18)$$

Is this the right definition of $\mathcal{D}\varphi^i(m)$? $\mathcal{D}\varphi^i(m)$ should be in the dual space of the functional tangent space in which the basis vectors are $\partial/\partial\varphi^j(m')$. So $\mathcal{D}\varphi^i(m)$ is defined by defining its action

$$\langle\mathcal{D}\varphi^i(m)|\frac{\partial}{\partial\varphi^j(m')}\rangle=\delta^i_j\delta(m-m') \qquad (19)$$

Inserting a product of n of these delta-functions in the action, we obtain

$$S[\varphi]=\sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial^nS[\varphi_0]}{\partial\varphi^{i_1}(m_1)\partial\varphi^{i_2}(m_2)..\partial\varphi^{i_n}(m_n)}\langle\mathcal{D}\varphi^{i_1}(m_1)|\frac{\partial}{\partial\varphi^{j_1}(m'_1)}\rangle\cdots\langle\mathcal{D}\varphi$$

$$\cdot(\varphi^{i_1}(m'_1)-\varphi_0^{i_1}(m'_1))(\varphi^{i_2}(m'_2)-\varphi_0^{i_2}(m'_2))\cdots(\varphi^{i_n}(m'_n)-\varphi_0^{i_n}(m'_n))dm'_n \qquad (20)$$

Now define

$$\Delta\varphi = \int (\varphi^j(m') - \varphi_0^j(m'))\frac{\partial}{\partial\varphi^j(m')}dm' \qquad (21)$$

and then

$$S[\varphi] = \sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial^n S[\varphi_0]}{\partial\varphi^{i_1}(m_1)\partial\varphi^{i_2}(m_2)..\partial\varphi^{i_n}(m_n)}\mathcal{D}\varphi^{i_1}(m_1)\otimes\cdots\mathcal{D}\varphi^{i_n}(m_n)\Delta\varphi\otimes \qquad (22)$$

$$= \sum_{0}^{\infty}\delta^{\otimes}S(\Delta\varphi)^{\otimes n}$$

Let us consider a symmetry under a group of bundle maps f:

$$f : E \to E \quad \text{bijective; and } f\circ\pi = \pi\circ f \quad \text{is a requirement for bundle map} \qquad (23)$$

This induces a transformation on $\mathcal{M}$, $\tilde{f}(\mathcal{M}) \to \mathcal{M}$ so that if

$$\pi \circ f(e) = \tilde{f}(e) \Leftarrow e \in F(m) \qquad (24)$$

$$\tilde{f}(m) = m', \text{then if } \pi \circ f(e) = m' \Leftrightarrow \pi(e) = m, \quad (25)$$

the symmetry transforms $\varphi_1 \rightarrow \varphi_2$ where

- $f : E \rightarrow E$
- $\tilde{f} : \mathcal{M} \rightarrow \mathcal{M}$ (defined from f when bundle map).
- $\pi \circ f = \tilde{f} \circ \pi$,

i.e. $f \circ \pi = \pi \circ f \Rightarrow$ the fiber F over say $\pi^{-1}(m)$ is mapped onto/into itself. For $e \in \pi^{-1}(m), \pi \circ f(e) = f \circ \pi(e)$ is independent of e, except through m. So for each m there is a map $f_m(e)$ inside the fiber on m,

$$\varphi_2(e) = f_{\pi(e)}(\varphi_1(\tilde{f}^{-1}(\pi(e)))) \quad (26)$$

A true diffeomorphism is defined by choosing an $\tilde{f}$ rather than f, and then deduce a f according to semi-local rules like

$$g^{\sigma\tau} \rightarrow \frac{\partial x^\sigma}{\partial x^\nu} \frac{\partial x^\tau}{\partial x^\mu} g^{\nu\mu} \quad (27)$$

This is semi-local: it only depends on derivatives and values near or at x, and then put them at x. Then we can almost choose $\tilde{f}$ freely and still get $\partial \mathrm{x}^\nu / \partial \mathrm{x}^\rho$, etc.

Assuming:

- that we have so much symmetry that all diffeomorphic maps $\tilde{f} : \mathcal{M} \to \mathcal{M}$ are achievable.
- that the full transformation $f : E \to E$ as far as the moving around on the fiber is concerned, i.e. f_m for all $m \in \mathcal{M}$, is determined by derivations of $\tilde{f}$ in the neighbourhood of m,

then we can prove that we can choose some subset of $\tilde{f}$'s in the supposed symmetry group so that it follows that

$$\frac{\partial^n S}{\partial \varphi^{i_1}(m_1) \partial \varphi^{i_2}(m_2) \cdots \partial \varphi^{i_n}(m_n)} \tag{28}$$

must be the same even if one moves any of the m_i's, except if this m_i coincides with (up to infinitesimals) another m, say m_j.

This implies first that if we ignore any grouping of the m_i, i.e. if they are all different, then (28) is independent of the m_i's. We should and could (if we think of true diffeomorphisms with usual tensors) also assume that

- we can arrange $\tilde{f}$ in such a way that the subsequent f can locally "rotate" or "transform" indices (on the φ's) so that only pseudoscalar not transformed so that only dependence on $\varphi^i(m_i)$ gets allowed.

So

$$\frac{\partial^n S}{\partial \varphi^{i_1}(m_1)\partial \varphi^{i_2}(m_2)\cdots \partial \varphi^{i_n}(m_n)} \tag{29}$$

is only allowed of the form $F(\varphi_0^{i_1}|_{scalar}, \cdots \varphi_0^{i_n}|_{scalar})$ Here the scalars are the $\varphi_0^{i_j}$-values corresponding to scalar components of the $\varphi^{i_j}(m)$. If so, only $\mathcal{D}\varphi^i(m)$ with a scalar component will be relevant in

$$\partial^{\otimes n} S = \int\int \frac{\partial^n S}{\partial\varphi^{i_1}(m_1)\cdots\partial\varphi^{i_n}(m_n)} \mathcal{D}\varphi^{i_1}(m_1)\cdots\mathcal{D}\varphi^{i_n}(m_n) dm_1 \cdots dm_n =$$

$$= \partial^{\otimes n} S|_{(\text{projected onto the "scalar" component})} \int \mathcal{D}\varphi^{i_1(scalar)}(m_1)\cdots\mathcal{D}\varphi^{i_n(scalar)}$$

(30)

$$= V^n F(\varphi_0^{i_1}|_{sc}, \varphi_0^{i_2}|_{sc}, \cdots \varphi_0^{i_n}|_{sc})$$

It is really not right to call the surviving components "scalar" unless we as in our old paper restrict to the $dx^1 \wedge dx^2 \wedge \cdots dx^n \wedge$ preserving diffeomorphism. But if we call them "pseudoscalars" it is OK.
We should moreover deduce that for all m_i being different,

$$\frac{\partial^n S}{\partial\varphi^{i_1}(m_1)\partial\varphi^{i_2}(m_2)\cdots\partial\varphi^{i_n}(m_n)} \quad (31)$$

can only depend on n and φ_0 and on the $i_1, i_2, \cdots i_n$ provided that the i_k's have "pseudoscalar" values. But if you have several "pseudoscalar" components i_k, (31) can depend on many of each. If for example we have 2 "pseudoscalar" i-values, we only get the integrals

$$\int \mathcal{D}\varphi^i(\varphi^i(n) - \varphi^i_0(m)) = V^i(\text{only i=1,2}) \tag{32}$$

where i is "pseudoscalar". Now our Taylor expansion takes the form

$$S[\varphi] = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{p=0}^{n} \binom{n}{p} f_n, p(V^1)^{n-p}(V^2)^p = F(V^1, V^2, \varphi_0) \tag{33}$$

where V^i's only depend on "pseudoscalar" components of $\varphi^i(m)$. We shall probably think of $\varphi_0 = 0$, but it may not matter.

There are also the cases where two or more of the m_i's are infinitely close/coinciding. In such cases we however only get non-negligible contributions to $S[\varphi]$ if we let the derivative

$$\frac{\partial^n S}{\partial\varphi^{i_1}(m_1)\partial\varphi^{i_2}(m_2)\cdots\partial\varphi^{i_n}(m_n)} \tag{34}$$

have factors $\delta(m_i - m_j)$. Derivatives of δ-functions may also contribute, then the derivative (34) will have a series of terms classified by clusterings of the m_i's. The number of ways of creating clusters corresponds to the partition $n = p_1 + p_2 + \cdots + p_l$ is

$$\binom{n}{p_1, p_2, \cdots, p_l} = \frac{n!}{p_1!p_2!\cdots, p_l!} \tag{35}$$

For each cluster with a number of say p m_i-values in it, we need a δ-function with $p-1$ delta functions $\delta(m_i - m_j)$, so as to compensate $p-1$ of the dm_i integrations, so that only one integration remains and gives us an all over the spacetime manifold $\mathcal{M}$ integral

$$\frac{\partial^n S}{\partial\varphi^{i_1}(m_1)\partial\varphi^{i_2}(m_2)\cdots\partial\varphi^{i_n}(m_n)}\Pi_{(\text{p-1 of the i-values})}\delta(m_i-m_j)\Pi_{(\text{p of the i-value}} \quad (36)$$

The invariance under transformations that only transform f in the neighbourhood of one of the clusters will only allow a non-zero contribution when the δ-functions of the cluster with the associated derivatives in the δ-functions run out to extract at the end of a "pseudoscalar" component from the (order p) product of the

$$(\varphi^{i_k}(m_k)-\varphi_0^{i_k}(m_k))(\varphi^{i_l}(m_k)-\varphi_0^{i_l}(m_k))\cdots \quad (37)$$

product it is going to multiply.
So apart the φ_0-term (though it best to just assume φ_0=0), the only non-zero cluster-contributions are total spacetime integrals

over "pseudoscalar" combinations of the fields, such as

$$\int \sqrt{g}(m) g^{\mu\nu}(m) \partial_\mu \varphi(m) \partial_\nu \varphi(m) dm \tag{38}$$

Here we could think of $\sqrt{g}$ as just a (fundamental) pseudoscalar field transforming under diffeomorphism symmetry with a determinant of the transformation partial derivatives,

$$\sqrt{g}(\mathrm{x}) \rightarrow det(\frac{\partial \mathrm{x}^\nu}{\partial \mathrm{x}^\mu}) \sqrt{g}(\mathrm{x}') \tag{39}$$

arranged in such a way that $\int \sqrt{g} dm$ is diffeomorphism invariant.
In a sufficiently complicated system of fields one can of course construct a lot of such pseudoscalar combinations like our example with "$\sqrt{g}$" or "$\sqrt{g}$" $g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$, or even something like $\epsilon^{\mu\nu\rho\sigma} \partial_\mu \varphi^1 \partial_\nu \varphi^2 \partial_\rho \varphi^3 \partial_\sigma \varphi^4$ (Guendelman).
Let us enumerate all these "pseudoscalars",

1. $P_1 =$ "$\sqrt{g}$"
2. $P_2 =$ "$\sqrt{g}$" $g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$
3. $P_3 = \epsilon^{\mu\nu\rho\sigma}\partial_\mu\varphi^1\partial_\nu\varphi^2\partial_\rho\varphi^3\partial_\sigma\varphi^4$
4. $P_4 = \cdots$

Everything in the Taylor expansion after choosing φ_0 (by field redefinition) becomes expressed by means of all integrals over $\mathcal{M}$ of the type $V^k = \int_\mathcal{M} P_k dm$. For sufficiently high n, we can expect to get the same P_k out of several of the clusters into which we partition such "big enough" n. In that case we might evaluate the

$$\frac{1}{n!}\begin{pmatrix} n \\ P_1 \cdots p_l \end{pmatrix} \tag{40}$$

and count the possibilities, but it is not really needed because the weight coefficients for the term combination can only be obtained if we somehow know the fundamental action functional S. We have already seen that we apriori shall get a series of terms in which all powers and all products of such powers of the integrals $V^k = \int_{\mathcal{M}} P_k dm$ occur. That is to say, we get an expression of the form

$$S[\varphi] = \sum_{k_1, k_2, \cdots k_q}^{\infty} C_{k_1 k_2 \cdots k_q} V^{k_1} V^{k_2} \cdots V^{k_q} \tag{41}$$

which in fact is the Taylor expansion for any function in the variables $(V^1, V^2, ..)$, provided one chooses the $C_{k_1 k_2 \cdots k_q}$ appropriately.

So all we have derived is that $S[\varphi]$ is a function of these variables $(V^1, V^2, ..)$, but we do not know which function. The variables on which are all $\mathcal{M}$-integrals of the "pseudoscalar" field combinations V^k. We however follow our earlier work where we derived an effective locality.

The main use of the action is via the Euler-Lagrange equations. Suppose we have a field ξ which can even be a component of some tensor field, or whatever; then the Euler-Lagrange equation for ξ is

$$\frac{\delta S[\varphi]}{\delta \xi(\mathrm{x})} = 0 \tag{42}$$

and now, since we derived S to be of the form $S[V^1, V^2, ..]$, we get

$$\frac{\delta S[\varphi]}{\delta \xi(\mathrm{x})} = \sum_k \frac{\partial S[\varphi]}{\partial V^k} \frac{\delta V^k}{\delta \xi(\mathrm{x})} = \tag{43}$$

$$= \sum_k \frac{\partial S[\varphi]}{\partial V^k} \frac{\delta \mathcal{P}_k}{\delta \xi(\mathrm{x})} (\text{mod partial integration}) = \frac{\delta S_{eff}}{\delta \xi(\mathrm{x})} \tag{44}$$

where

$$S_{eff} = \int_{\mathcal{M}} \sum_k \frac{\partial S}{\partial V^k} \mathcal{P}_k(\mathrm{x}) dm \tag{45}$$

which by construction is local, provided the coefficients $\partial S / \partial V^k$ do not depend on the V^k's. But these V^k are "constants" in the

sense that they do not depend on space and time, i.e. not on $m \in \mathcal{M}$ (careful with double labeling with m and x).
So we got locality except that the coupling constants via the $\partial S/\partial V^k$'s depend on integrals taken all over spacetime.

Some remarks on random dynamics, H.B. Nielsen and N. Brene Proc. of the 2nd Nishinomiya Yukawa Memorial Symposium on String Theory, Kyoto University, 1987 (Springer, Berlin, 1988); Random Dynamics and relations between the number of fermion generations and the fine structure constants, H. B. Nielsen (Bohr Inst.) NBI-HE-89-01, Jan 1989. 50pp. Talk presented at Zakopane Summer School, May 41 - Jun 10, 1988. Published in Acta Phys.Polon.B20:427, 1989; Origin of Symmetries, C. D. Froggatt and H. B. Nielsen, World Scientific (1991).

- H. Bech Nielsen and A. Kleppe, Towards a Derivation of Space, 1403.1410v1, Proceedings to the 16th Workshop "What Comes Beyond the Standard Models?", Bled 14-21 of July 2013, Vol. 14.
- H. Bech Nielsen and A. Kleppe, Deriving diffeomorphism symmetry, 1412.7497v1 gr-qc, Bled Workshops Phys. 15 (2014) no.2, 267-282.
- D. L. Bennett, C. D. Froggatt, H. B. Nielsen, Nonlocality as an explanation for finetuning and field replication in nature, NBI-95-15, GUTPA/95/04/1.